EDO-EP-35

December, 2000

Localization of Bulk Fields on AdS_4 Brane in AdS_5

Ichiro Oda [1]

Edogawa University, 474 Komaki, Nagareyama City, Chiba 270-0198, JAPAN

Abstract

We study the localization of various bulk fields on an AdS_4 brane in AdS_5. In this case, for a small brane cosmological constant, all the bulk fields ranging from scalar to graviton are naturally confined to the brane only through the gravitational interaction. In particular, for the vector field we can find an interesting zero mode solution which satisfies the box boundary condition. In the cases of spin 1/2 spinor and 3/2 gravitino fields, the form of zero modes is very similar to as in a flat Minkowski brane, but they are trapped on the AdS_4 brane even without introducing a mass term with a 'kink' profile.

[1]E-mail address: ioda@edogawa-u.ac.jp

The past few years have witnessed a lot of interest in the study of a brane world where the standard model is assumed to live on a brane embedded in a higher dimensional space-time whereas the gravitational field is free to propagate in the whole space-time. Contact with observations obviously requires that the four dimensional gravitational theory is not only reproduced on the brane but also the standard model particles are confined on the brane by some ingenious mechanism. The most interesting possibility is provided by superstring theory or M-theory, but it is at present far from complete to understand the precise procedure of deriving a desired model of the brane world from superstring theory even if there have been many efforts and attempts so far. Then it is a natural step to pursue a different possibility where the local field theory gives us such a mechanism.

Indeed, Randall and Sundrum have found a solution to the five dimensional Einstein's equations with a Minkowski flat 3-brane in AdS_5 and shown that the effects of the four dimensional gravity on the brane is reproduced without the need to compactify the fifth dimension [1, 2]. (This model is generalized to the case of many branes in Ref. [3, 4].) Regarding the localization of the standard model particles, it has been also shown that the local field theory can present a nice mechanism in an arbitrary space-time dimension [5].

Recently, Karch and Randall have found a unexpected result that four dimensional gravity is induced even on an AdS_4 brane in an AdS_5 background owing to the locaization of a massive and normalizable bound state [6]. (See also related works [7, 8, 9, 10, 11].) In this model, in order to avoid the conflict with experiment, a four dimensional cosmological constant is taken to be small in Planck units, so the massive bound state is effectively massless at large scales. Then, a question, which naturally arises, is whether or not the standard model particles are also localized on the AdS_4 brane in terms of the gravitational interaction as in a Minkowski 3-brane. This is precisely the question with which we are concerned in this article. We will see below that the whole particles are trapped on the brane in a remarkable way.

Let us first fix our physical setup. Following the work of Karch and Randall we shall consider a five dimensional anti-de Sitter space-time (AdS_5) with a warp factor

$$ds^2 = e^{2A(z)} \left(\hat{g}_{\mu\nu}(x^\lambda) dx^\mu dx^\nu + dz^2 \right), \tag{1}$$

where $\mu, \nu, \lambda = 0, 1, 2, 3$. Here the metric $\hat{g}_{\mu\nu}$ on a 3-brane denotes an AdS_4 background. Moreover, the warp factor $A(z)$ in the conformal coordinate z is given by

$$e^{A(z)} = \frac{L\sqrt{-\Lambda}}{\sin\sqrt{-\Lambda}(|z| + z_0)}, \tag{2}$$

with the five dimensional cosmological constant Λ_5 being connected with a constant L by $\Lambda_5 = -\frac{3}{L^2}$. Now the four dimensional cosmological constant Λ is assumed to be small and negative. In this article, we follow the standard conventions and notations of the textbook of Misner, Thorne and Wheeler [12]. Also note that a single positive tension brane sits at the origin $z = 0$ and orbifold boundary conditions are imposed in a usual way. From the form of $e^{A(z)}$ and orbifold boundary conditions, we can limit ourselves to the region $0 < z < \frac{\pi}{\sqrt{-\Lambda}} - z_0$. Instead of the z coordinate, we sometimes make use of the coordinate w whose definition is given by $w \equiv \sqrt{-\Lambda} z$ (and $\varepsilon \equiv \sqrt{-\Lambda} z_0$), so in the w coordinate $0 < w < \pi - \varepsilon$. Since we have fixed our physical setup, in what follows, we shall study the property of localization of bulk fields according to their spin in order. As a remark, in this article, we will assume that the background metric is not modified by the presence of the bulk fields, namely, we will neglect the back-reaction on the metric from the bulk fields.

We are now ready to start with the case of a massless real scalar field of spin 0. The extension to complex and/or massive scalar fields is straightforward. The action takes the form

$$S_0 = -\frac{1}{2} \int d^5 x \sqrt{-g} g^{MN} \partial_M \Phi \partial_N \Phi, \tag{3}$$

where M, N denote the five dimensional space-time indices. From this action, we have the equation of motion, $\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) = 0$. With the metric ansatz (1), this equation of motion leads to

$$\left[-\partial_z^2 + \frac{9}{4} A'(z)^2 + \frac{3}{2} A''(z) \right] \tilde{\Phi} = m^2 \tilde{\Phi}, \tag{4}$$

where we have defined $\tilde{\Phi}$ as $\tilde{\Phi} \equiv e^{\frac{3}{2} A(z)} \Phi$ and assumed that $\Box \tilde{\Phi} \equiv m^2 \tilde{\Phi}$ with the definition of 4D d'Alembertian operator $\Box \equiv \frac{1}{\sqrt{-\hat{g}}} \partial_\mu (\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu)$. The prime denotes differentiation with respect to the coordinate z. Eq. (4) can be cast to the Schrödinger-like equation in the w coordinate with help of Eq. (2) as follows:

$$\left[-\partial_w^2 + V(w) \right] \tilde{\Phi} = E \tilde{\Phi}, \tag{5}$$

where $E \equiv |\Lambda| m^2$ and the potential $V(w)$ is of the form

$$V(w) = -\frac{9}{4} + \frac{15}{4} \frac{1}{\sin^2(|w| + \varepsilon)} - 3 \cot(\varepsilon) \delta(w). \tag{6}$$

In the case where $\varepsilon = \frac{\pi}{2}$, the delta function decouples, thereby implying that there is no brane, so we can find a general solution in terms of the hypergeometric function

$$\begin{aligned} \tilde{\Phi}(w) &= A_1 \frac{1}{(\sin w)^{\frac{3}{2}}} F(-\frac{3}{4} + \frac{\sqrt{4E+9}}{4}, -\frac{3}{4} - \frac{\sqrt{4E+9}}{4}, \frac{1}{2}, \cos^2 w) \\ &+ A_2 \frac{\cos w}{(\sin w)^{\frac{3}{2}}} F(-\frac{1}{4} + \frac{\sqrt{4E+9}}{4}, -\frac{1}{4} - \frac{\sqrt{4E+9}}{4}, \frac{3}{2}, \cos^2 w), \end{aligned} \tag{7}$$

where A_1, A_2 are integration constants. To gain a solution to Eq. (6), one needs to impose the boundary conditions at $w = 0$ and $w = \pi - \varepsilon$, but it turns out that it is a delicate problem and consequently only the numerical analysis is available [6].

In this respect, we make use of an exactly solvable toy model sharing the similar qualitative features with the original model (5) [10]. In the toy model, the potential takes the form

$V(w) = -3\cot(\varepsilon)\delta(w)$ and contains a wall at $w = \pi - \varepsilon$ [10]. Then there is a bound state solution $\chi_0(w)$ with energy $E = -\kappa^2$

$$\chi_0(w) = \sinh\kappa(\pi - \varepsilon - w) \tag{8}$$

where κ must satisfy the equation

$$\frac{2}{3}\tan\varepsilon = \frac{1}{\kappa}\tanh\kappa(\pi - \varepsilon) \tag{9}$$

In addition to this bound state, it is easy to find a set of massive modes with energy $E = k^2$, which are given by

$$\chi_k(w) = \sin k(\pi - \varepsilon - w) \tag{10}$$

where k must satisfy the equation

$$\frac{2}{3}\tan\varepsilon = \frac{1}{k}\tan k(\pi - \varepsilon) \tag{11}$$

These results are similar to those of the gravitational field [6, 10]. Actually it is known that transverse traceless graviton modes in general obey the equation of a massless scalar in a curved background, so the results obtained here merely confirms this fact in an explicit manner. But owing to the existence of the brane cosmological constant there is a slight differece between the two cases, which is that in the case at hand $\Box\tilde{\Phi} = m^2\tilde{\Phi}$ while in the gravity case $(\Box + 2|\Lambda|)h_{\mu\nu}^{TT} = m^2 h_{\mu\nu}^{TT}$.

Let us turn our attention to the localization of a scalar field on an AdS_4 brane in AdS_5. In the situation at hand, let us focus on only the bound state (8) because this bound state corresponds to the massive graviton mode and the treatment of a set of massive modes can be done in an analogous way. Following the method [13, 14], let us plug the bound state solution $\Phi_0(x^M) = \phi(x^\mu)\chi_0(z)e^{-\frac{3}{2}A(z)}$ into the starting action:

$$\begin{aligned} S_0^{(0)} &= -\frac{1}{2}\int d^5x\sqrt{-g}g^{MN}\partial_M\Phi_0\partial_N\Phi_0 \\ &= -\frac{1}{2}\int d^4x\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz\chi_0^2(z). \end{aligned} \tag{12}$$

Here it is worthwhile to notice that the condition that the bound state solution is localized on a brane is equivalent to the normalizability of the state wave function on the brane [13, 14]. This condition now amount to the finiteness of an integral over z coordinate in Eq. (12). Indeed, we can show that the integral is finite as follows:

$$\begin{aligned} I_0 &\equiv \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz\chi_0^2(z) \\ &= \frac{1}{4\kappa\sqrt{-\Lambda}}\left[\sinh 2\kappa(\pi - \varepsilon) - 2\kappa(\pi - \varepsilon)\right] < \infty. \end{aligned} \tag{13}$$

Note that I_0 is strictly finite as long as the brane cosmological constant Λ is nonzero. Thus the bound state of scalar field is confined on an AdS_4 brane in AdS_5 by the gravitational interaction. This proof obviously means that transverse, traceless, massive modes of the gravitational field are also confined to the brane.

Next let us turn to spin 1/2 spinor field. The starting action is the conventional Dirac action with a mass term:

$$S_{1/2} = \int d^5x\sqrt{-g}\bar{\Psi}i(\Gamma^M D_M + m\varepsilon(z))\Psi, \tag{14}$$

where the covariant derivative is defined as $D_M\Psi = (\partial_M + \frac{1}{4}\omega_M^{AB}\gamma_{AB})\Psi$ with the definition of $\gamma_{AB} = \frac{1}{2}[\gamma_A, \gamma_B]$, and $\varepsilon(z)$ is $\varepsilon(z) \equiv \frac{z}{|z|}$ and $\varepsilon(0) \equiv 0$. Here the indices A, B are the ones of the local Lorentz frame and the gamma matrices Γ^M and γ^A are related by the vielbeins e_A^M through the usual relations $\Gamma^M = e_A^M\gamma^A$ where $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$ and $\{\gamma^A, \gamma^B\} = 2\eta^{AB}$. A feature of the action is the existence of a mass term with a 'kink' profile. We have just introduced this type of mass term in the action since the existence has played a critical role in the localization of fermionic fields on a Minkowski brane in an arbitrary dimension [5]. However, as will be shown in what follows, in the case of an AdS brane such the mass term does not play any important role. This is one of interesting things in the model at hand compared to a flat Minkowski brane.

In the background (1), the torsion-free conditions yield an explicit expression of the spin connections:

$$\omega_\mu = \frac{1}{2}A'(z)\gamma_\mu\gamma_z + \hat{\omega}_\mu(\hat{e}),\ \omega_z = 0, \tag{15}$$

where we have defined $\omega_M \equiv \frac{1}{4}\omega_M^{AB}\gamma_{AB}$, $\hat{\omega}_\mu(\hat{e}) \equiv \frac{1}{4}\hat{\omega}_\mu^{ab}(\hat{e})\gamma_{ab}$, and $\gamma_\mu \equiv \hat{e}_\mu^a\gamma_a$. Using Eq. (15), the Dirac equation $(\Gamma^M D_M + m\varepsilon(r))\Psi = 0$ stemming from the action (14) can be cast to the form

$$[\Gamma^z(\partial_z + 2A') + m\varepsilon(r) + \Gamma^\mu(\partial_\mu + \hat{\omega}_\mu)]\,\Psi = 0. \tag{16}$$

Let us find the massless zero-mode solution with the form of $\Psi(x^M) = \psi(x^\mu)u(z)$ such that $\Gamma^\mu\hat{D}_\mu\psi \equiv \Gamma^\mu(\partial_\mu + \hat{\omega}_\mu)\psi = 0$ and the chirality condition $\Gamma^z\psi = \psi$ is imposed on the brane fermion. Then, Eq. (16) is reduced to a first-order differential equation to $u(z)$ and is easily integrated to be

$$u(z) = u_0 e^{-2A(z) - m\varepsilon(z)z}, \tag{17}$$

with an integration constant u_0.

In order to check the localization of this mode, let us insert this solution into the Dirac action (14). Then the action reduces to the form

$$\begin{aligned} S_{1/2}^{(0)} &= \int d^5x\sqrt{-g}\bar{\Psi}^{(0)}i(\Gamma^M D_M + m\varepsilon(z))\Psi^{(0)} \\ &= \int d^4x\sqrt{-\hat{g}}\bar{\psi}i\gamma^\mu\hat{D}_\mu\psi \int_0^{\frac{\pi}{\sqrt{-\Lambda}} - z_0} dz e^{4A(z)}u^\dagger(z)u(z) + \cdots. \end{aligned} \tag{18}$$

Again the condition of the trapping of the bulk spinor on an AdS_4 brane requires that an integral over z has a finite value. The integral is easily evaluated as follows:

$$\begin{aligned} I_{1/2} &\equiv \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{4A(z)} u^\dagger(z) u(z) \\ &= |u_0|^2 \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{-2mz} < \infty. \end{aligned} \tag{19}$$

Here one interesting thing has happened. Recall that in a Minkowski brane, only the massive bulk fermion with a 'kink' profile is localized on the brane whereas the massless one is not so. This fact can be traced in Eq. (19) since $I_{1/2}$ at $\Lambda = 0$ is divergent in the massless limit $m = 0$. (Note that in both the Minkowski brane and the AdS brane, the form of the zero-mode solution of fermion, (17), is common so this consideration is legitimate.) In the case at hand, irrespective of the presence of mass term, the bulk spinor can be localized on the brane through the gravitational interaction as long as the brane cosmological constant is nonvanishing.

Now we are willing to consider spin 1 $U(1)$ vector field. Incidentally the generalization to the nonabelian gauge fields is straightforward. And the inclusion of bulk mass does not change the results obtained below. The action reads

$$S_1 = -\frac{1}{4} \int d^5x \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS}, \tag{20}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. The equations of motion $\partial_M(\sqrt{-g} g^{MN} g^{RS} F_{NR}) = 0$ can be solved under the gauge condition $A_z = 0$. The solution with the form $A_\mu(x^M) = a_\mu(x^\lambda)\rho(z)$ is searched where a_μ satisfy the equations $\hat{\nabla}^\mu a_\mu = \partial^\mu f_{\mu\nu} = 0$ with the definition of $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$. Then, the Maxwell equations are reduced to a single differential equation:

$$\partial_z(e^{A(z)} \partial_z \rho(z)) = 0. \tag{21}$$

In the case of a Minkowski brane, we have selected a constant zero-mode solution $\rho(z) = const$, which leads to non-localization of the vector field. On the other hand, in an AdS brane a new solution is available, which is given by $e^{A(z)} \partial_z \rho(z) = const \neq 0$. (Note that this solution is not localized on a Minkowski brane, either.) As a result, we obtain a solution to Eq. (21):

$$\rho(z) = -\frac{\alpha}{L(-\Lambda)} \cos \sqrt{-\Lambda}(z + z_0) + \beta, \tag{22}$$

where α, β are integration constants.

Let us investigate whether this solution is localized on an AdS_4 brane or not according to the method used above. The substitution of this solution into the action leads to

$$\begin{aligned} S_1^{(0)} &= -\frac{1}{4} \int d^5x \sqrt{-g} g^{MN} g^{RS} F_{MR}^{(0)} F_{NS}^{(0)} \\ &= -\frac{1}{4} \int d^4x \sqrt{-\hat{g}} \hat{g}^{\mu\nu} \hat{g}^{\lambda\sigma} f_{\mu\lambda} f_{\nu\sigma} \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{A(z)} \rho^2(z) \\ &\quad -\frac{1}{4} \int d^4x \sqrt{-\hat{g}} \hat{g}^{\mu\nu} a_\mu a_\nu \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz \ 2e^{A(z)} (\partial_z \rho(z))^2. \end{aligned} \tag{23}$$

Here we have carefully kept the KK-mass term since we wish to examine whether this solution leads to massless 'photon' on a brane. The localization condition of this mode on a brane requires the first integral over z to be finite. Thus let us focus on this integral first.

$$
\begin{aligned}
I_1^{(1)} &\equiv \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{A(z)} \rho^2(z) \\
&= \int_0^{\pi-\varepsilon} dw \frac{L}{\sin(w+\varepsilon)} \left[\frac{\alpha}{L\Lambda} \cos(w+\varepsilon) + \beta \right]^2 .
\end{aligned} \tag{24}
$$

This integral $I_1^{(1)}$ is in general divergent, but only when the equality $\frac{\alpha}{L\Lambda} = \beta$ holds, it becomes to be finite. Henceforth, we shall consider this specific case. Then, it is straightforward to calculate the above integral as well as the second integral over z in Eq. (23) associated with the KK-mass term whose result is given by

$$
\begin{aligned}
S_1^{(0)} &= -\frac{1}{4} \int d^4x \sqrt{-\hat{g}} \hat{g}^{\mu\nu} \hat{g}^{\lambda\sigma} f_{\mu\lambda} f_{\nu\sigma} \left(-\frac{2\alpha^2}{L\Lambda^2}\right) \left[\cos^2 \frac{\varepsilon}{2} + \log(\sin^2 \frac{\varepsilon}{2}) \right] \\
&\quad -\frac{1}{4} \int d^4x \sqrt{-\hat{g}} \hat{g}^{\mu\nu} a_\mu a_\nu \frac{4\alpha^2}{L(-\Lambda)} \cos^2 \frac{\varepsilon}{2}.
\end{aligned} \tag{25}
$$

The quantities in front of the kinetic and the mass terms are obviously finite, so the gauge field is localized on an AdS_4 brane, which is contrasted with the case of a Minkowski brane [5].

At this stage, we can take a step further. Namely, provided that when $\varepsilon \approx 0$ we redefine the brane gauge field a_μ as

$$
\sqrt{-\frac{2\alpha^2}{L\Lambda^2} \left[\cos^2 \frac{\varepsilon}{2} + \log(\sin^2 \frac{\varepsilon}{2}) \right]} \, a_\mu \to a_\mu, \tag{26}
$$

Eq. (25) reads

$$
S_1^{(0)} = -\frac{1}{4} \int d^4x \sqrt{-\hat{g}} \left[\hat{g}^{\mu\nu} \hat{g}^{\lambda\sigma} f_{\mu\lambda} f_{\nu\sigma} + 2\Lambda \frac{\cos^2 \frac{\varepsilon}{2}}{\cos^2 \frac{\varepsilon}{2} + \log(\sin^2 \frac{\varepsilon}{2})} \hat{g}^{\mu\nu} a_\mu a_\nu \right]. \tag{27}
$$

This expression implies that the mass of the brane gauge field is very tiny if the brane cosmological constant $|\Lambda| \approx \varepsilon^2$ is small enough. In the work of Karch and Randall [6] the small brane cosmological constant is needed to make contact with experiment. In the present analysis the smallness of the brane cosmological constant is obtained from the physical requirement that the $U(1)$ gauge field a_μ must be *massless* 'photon' on an AdS_4 brane. As a final remark in this paragraph, let us notice that owing to the equality $\frac{\alpha}{L\Lambda} = \beta$, our solution (22) reduces to the form

$$
\rho(z) = -\frac{2\alpha}{L(-\Lambda)} \cos^2 \frac{\sqrt{-\Lambda}}{2}(z + z_0). \tag{28}
$$

Like the graviton as well as a real scalar, this solution satisfies the box boundary condition at $z = \frac{\pi}{\sqrt{-\Lambda}} - z_0$, where $\rho(z) = 0$. It is quite of interest that the requirement of the localization for the gauge field naturally leads to the same boundary condition as the other bosonic fields.

Finally, let us consider the gravitino field of spin 3/2. It is well known that even if free field actions for any higher spin do indeed exist, we cannot construct interacting actions for more than spin 2 at least within the framework of the local field theory. Thus, it is now sufficient to consider the remaining spin 3/2 case for the study of localization of the whole spin fields. (As mentioned before, transverse traceless graviton modes of spin 2 have the same localization property as a real scalar. And the higher-rank antisymmetric tensor fields can be treated in a similar way to the electro-magnetic field.) The trapping of the gravitino might be automatic when the graviton is trapped and the theory is supersymmetrized since the gravitino is anyway a superpartner of the graviton, but at the present time of writing this article we do not have a grasp of a supersymmetric theory corresponding to Karch-Randall model, so it is valuable to pursue the issue along the same line of arguments as above.

The action for spin 3/2 bulk gravitino is given by the Rarita-Schwinger action [5]

$$S_{3/2} = \int d^5x \sqrt{-g}\bar{\Psi}_M i\Gamma^{[M}\Gamma^N\Gamma^{R]}(D_N + \delta^z_N \Gamma_z m\varepsilon(z))\Psi_R, \tag{29}$$

where $D_M\Psi_N = \partial_M\Psi_N - \Gamma^R_{MN}\Psi_R + \frac{1}{4}\omega_M^{AB}\gamma_{AB}\Psi_N$ and the square bracket denotes the anti-symmetrization with weight 1. From the metric condition $D_M e^A_N = \partial_M e^A_N - \Gamma^R_{MN}e^A_R + \omega_M^{AB}e_{NB} = 0$, we obtain the concrete expression of the affine connections $\Gamma^R_{MN} = e^R_A(\partial_M e^A_N + \omega_M^{AB}e_{NB})$. In the background (1), the affine connections are calculated to be

$$\begin{aligned}
\Gamma^\rho_{\mu\nu} &= \hat{\Gamma}^\rho_{\mu\nu}(\hat{e}),\ \Gamma^\rho_{\mu z} = \Gamma^\rho_{z\mu} = A'(z)\delta^\rho_\mu, \\
\Gamma^z_{\mu\nu} &= -A'(z)\hat{g}_{\mu\nu},\ \Gamma^z_{zz} = A'(z),\ otherwise = 0.
\end{aligned} \tag{30}$$

With the gauge condition $\Psi_z = 0$, the nonvanishing components of $D_M\Psi_N$ read

$$\begin{aligned}
D_\mu\Psi_\nu &= \hat{D}_\mu(\hat{e})\Psi_\nu + \frac{1}{2}A'(z)\gamma_\mu\gamma_z\Psi_\nu, \\
D_\mu\Psi_z &= -A'(z)\Psi_\mu, \\
D_z\Psi_\mu &= (\partial_z - A'(z))\Psi_\mu,
\end{aligned} \tag{31}$$

where we have used Eqs. (15) and (30). And we have defined $\hat{D}_\mu(\hat{e})\Psi_\nu \equiv \partial_\mu\Psi_\nu - \hat{\Gamma}^\rho_{\mu\nu}\Psi_\rho + \hat{\omega}_\mu(\hat{e})\Psi_\nu$.

The equations of motion $\Gamma^{[M}\Gamma^N\Gamma^{R]}(D_N + \delta^z_N\Gamma_z m\varepsilon(z))\Psi_R = 0$ can be cast to the form

$$g^{\mu\nu}\left[\Gamma^z(\partial_z + A'(z)) + m\varepsilon(z)\right]\Psi_\nu = 0, \tag{32}$$

where we have used equations $\gamma^\mu\Psi_\mu = \hat{D}^\mu\Psi_\mu = \gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\hat{D}_\nu\Psi_\rho = 0$. Let us look for a solution with the form $\Psi_\mu(x^M) = \psi_\mu(x^\lambda)v(z)$. If the chirality condition $\Gamma^z\psi_\mu = \psi_\mu$ is utilized in Eq. (32), we can get a solution $v(z) = v_0 e^{-A(z)-m\varepsilon(z)z}$ in a perfectly analogous manner to the case of spin 1/2 spinor.

Substituting this solution into the action (29), we arrive at the following expression

$$
\begin{aligned}
S^{(0)}_{3/2} &= \int d^5x\sqrt{-g}\bar{\Psi}^{(0)}_M i\Gamma^{[M}\Gamma^N\Gamma^{R]}(D_N + \delta^z_N\Gamma_z m\varepsilon(z))\Psi^{(0)}_R \\
&= \int d^4x\sqrt{-\hat{g}}\bar{\psi}_\mu i\gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\hat{D}_\nu\psi_\rho \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{2A(z)}v^\dagger(z)v(z) + \cdots. \qquad (33)
\end{aligned}
$$

Again the condition for the localization of the gravitino on a brane requires the integral over z to take a finite value. Indeed this statement can be checked as follows:

$$
\begin{aligned}
I_{3/2} &\equiv \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{2A(z)}v^\dagger(z)v(z) \\
&= |v_0|^2 \int_0^{\frac{\pi}{\sqrt{-\Lambda}}-z_0} dz e^{-2mz} < \infty. \qquad (34)
\end{aligned}
$$

At this stage, it is of interest to notice that the condition has the same form as in spin 1/2 spinor field up to an irrelevant constant, so whenever spin 1/2 fermion is localized, spin 3/2 gravitino is also localized on a brane.

In conclusion, in this article we have presented a complete analysis of localization of all bulk fields on an AdS_4 brane in AdS_5 from the viewpoint of the local field theory. We can summarize the results obtained in this article as follows. It has been shown that the whole bulk fields can be localized on an AdS_4 brane by only the gravitational interaction without invoking additional interactions. In particular, for the localization of fermionic fields we do not have to introduce a mass term with a 'kink' profile into the action. Moreover, the existence of the massless gauge field on the brane requires that the brane cosmological constant should be small enough not to contradict the standard model.

Accordingly, it seems that Karch-Randall model [6] gives us an interesting phenomenological model. A direction for future study is to examine whether this model could also provide a realistic cosmological model [15]. Another interesting direction is to supersymmetrize the model at hand.

References

[1] L. Randall and R. Sundrum, Phys.Rev.Lett. **83** (1999) 3370, hep-th/9905221.

[2] L. Randall and R. Sundrum, Phys.Rev.Lett. **83** (1999) 4690, hep-th/9906064.

[3] I. Oda, Phys.Lett. **B480** (2000) 305, hep-th/9908104.

[4] I. Oda, Phys.Lett. **B472** (2000) 59, hep-th/9909048.

[5] I. Oda, "Concise Encyclopedia on Supersymmetry and Noncommutative Structures in Mathematics and Physics", edited by J. Bagger, S. Duplij and W. Siegel, Kluwer Academic Publishers, Dordrecht (in press), hep-th/0009074 and references therein.

[6] A. Karch and L. Randall, hep-th/0011156.

[7] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, hep-th/0011138.

[8] M. Porrati, hep-th/0011152.

[9] A. Miemiec, hep-th/0011160.

[10] M.D. Schwartz, hep-th/0011177.

[11] P.A. Grassi and P. van Nieuwenhuizen, hep-th/0011278.

[12] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, San Francisco: Freeman (1973).

[13] B. Bajc and G. Gabadadze, Phys.Lett. **B474** (2000) 282, hep-th/9912232.

[14] I. Oda, Phys.Lett. **B** (in press), hep-th/0006203; Phys.Rev. **D62** (2000) 126009, hep-th/0008012.

[15] I. Oda, to appear.